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SEQUENTIAL DECISIONMAKING: APPLICATIONS TO RETIREMENT AND WELFA--ETC(U)

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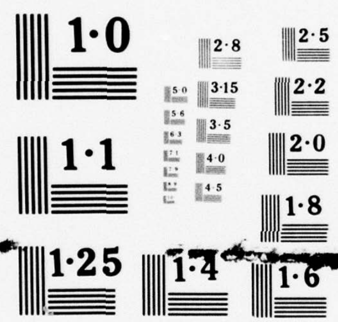

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SEQUENTIAL DECISIONMAKING:
APPLICATIONS TO RETIREMENT AND WELFARE PARTICIPATION

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I. INTRODUCTION

This paper is based on ongoing research in three different but related areas: the theory of search, optimal retirement policies, and participation in alternative welfare programs. The research in each area is joint--my co-workers being Steve Lippman, Glenn Gotz, and Jim Hosek, respectively.

In general, we are concerned with a life cycle model of optimal decisionmaking.[1] The life cycle can be decomposed into several stages including human capital formation and occupational choice, the working period where individuals are subject to layoffs and may find it desirable to quit and change job or occupation, and the retirement period.

Here I focus on two different groups whose members are in the working period of the life cycle. The first is a fairly homogeneous group consisting of Air Force officers who are deciding whether they should leave the Air Force (retire) and begin a new career in the civilian sector or remain in the Air Force. As we will see the structure of the Air Force retirement system is quite similar to that of the entire civil service and consequently the model developed here has much wider applicability.[2]

The second group of individuals consists of unemployed males who must choose between participation in unemployment compensation or the unemployed fathers welfare program. Of course, they may choose to search on their own receiving no income transfer. We assume that those who choose to participate in either transfer program are actively engaged in job search. Once again the model designed for this special group has wider applicability as it could be used to study the behavior of any unemployed worker when choosing among alternative income transfer programs.

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[1]See Heckman and references therein.

[2]For a thorough discussion of this model and its applications, see *The Retirement Decision: "Numerical Analysis of a Dynamic Model,"* by Gotz and McCall, (forthcoming).

II. OPTIMAL RETIREMENT POLICIES FOR AIR FORCE OFFICERS

1. INTRODUCTION

This section first presents an elementary model of retirement. It is probably the simplest version of a dynamic retirement model in which individuals need not consume full leisure after the retirement decision--retirement from the Air Force being more like a quit than a typical retirement.[1] The fact that individuals have opportunities for post-retirement employment is explicitly considered. The model also directly measures the supplement to post-Air Force income deriving from the pension that has been accrued at the retirement decision point. All of this is done in an infinite horizon setting, where the decision-maker has some uncertainty about his nonmilitary employment prospects, but knows for sure his military prospects. Uncertainty about civilian employment is summarized by a probability distribution of job offers which the decisionmaker does know with certainty. The choice of a job from this distribution is accomplished in optimal fashion using the standard sequential search paradigm. Once a civilian job is accepted, the individual remains there forever--the model does not permit quits or layoffs in the civilian market. Individuals are assumed to be risk neutral, the job offer distribution is insensitive to the business cycle, a single job offer arrives each period, the cost of search is constant per period, and there is discounting.

The basic choice is between remaining in the Air Force and entering the civilian labor market. Notice that entering the civilian labor market includes full leisure, retirement in the usual sense, as a special case. An individual will choose full leisure when it dominates all other civilian labor market opportunities *and* the alternative of remaining in the Air Force.

2. A SIMPLE RETIREMENT MODEL

We first consider the case where there are no promotions and an invariant wage offer distribution in the civilian sector. Let F denote the cumulative offer distribution; x, the wages in the civilian

[1] See Cotterman for analysis of typical retirements.

sector (F ranges over positive values of x); m, the known and constant wages in the military sector; $\beta \equiv (1/1 + \rho)$, the discount factor with ρ the interest rate; T, the transaction costs associated with transiting from the military to the civilian sector; and r, the pension parameter denoting the fraction of military pay that is collected per period after retirement, $0 \leq r \leq 1$.

In this setting it is easy to show that the optimal policy for an individual who has retired and is searching for employment in the civilian sector possesses the following structure:

$$\text{accept employment, if } x \geq \xi$$

$$\text{continue searching, if } x < \xi \quad ,$$

where ξ is the solution to:

$$\frac{\xi}{1 - \beta} = -c + \frac{\xi}{1 - \beta} \text{ E max } (\xi, x) \quad .$$

[Proof: Let V(x) denote the optimal return from search when an offer x has been received. Then

$$V(x) = \max \{x, - c + \int_0^\infty V(y)dF(y)\}$$

where the second term inside the braces is the expected return from continued search and x is return from stopping. Rewrite V(x) as

$$V(x) = \max (x, \xi) \text{ with}$$

$$\xi = -c + \int_0^\infty V(y)dF(y)$$

$$= -c + \text{E max } (x, \xi) \quad .$$

With discounting this becomes

$$c = \beta\{E \max (x, \xi) - \xi\} \quad .$$

Inserting the present value of the right-hand side (the expected marginal return from search) gives

$$c = \frac{\beta}{1 - \beta} (E \max (x, \xi) - \xi) \quad .]$$

Offers are received at the end of the period, whereas costs and wage payments occur at the beginning of each period. The reservation wage, ξ, is also the expected return from search when this policy is pursued. Consequently, in deciding whether to remain in the Air Force or retire to the civilian sector, the decisionmaker will compare the discounted return from retirement, $(\xi + rm/1 - \beta) - T$, with the discounted return from staying in, $m/1 - \beta$. (The transition cost, T, is incurred as soon as the decision to retire is made. Of course, it may be negative.) Thus, the optimal retirement policy has the following simple form:

$$\text{if } \frac{(1 - r)m}{1 - \beta} \leq \frac{\xi}{1 - \beta} - T \quad , \text{ retire}$$

$$\text{if } \frac{(1 - r)m}{1 - \beta} > \frac{\xi}{1 - \beta} - T \quad , \text{ stay in} \quad .$$

This can be restated in terms of r (assume $T = 0$):

$$\text{if } r \geq r^* \quad , \text{ retire}$$

$$\text{if } r < r^* \quad , \text{ stay in}$$

where $r^* = 1 - \xi/m$.

3. A RETIREMENT MODEL WITH PROMOTIONS

We now describe a more complex retirement model that includes the preceding elementary model as a special case. Let $i = 1, 2, 3, \ldots, k$; denote the k different grade levels associated with employment in the Air Force Officer Corps. For example, $i = 1$ represents a second lieutenant and $i = k$ represents a four-star general. Movement among these grades is assumed to be governed by a first order Markov chain with transition probabilities p_{ij}^{t}, $i = 1, \ldots, k$; $j = 1, \ldots, k$; $t =$ years of service. Thus p_{ij}^{t} is the probability of going to grade level j in the next period given that this period's grade level is i and years of service equals t.[1] Demotions are extremely rare in the Air Force so we assume that $p_{ij} = 0$ whenever $j < i$. This, of course, implies that the Markov matrix P of transition probabilities is upper triangular. The upper triangular portion of the Markov matrix is also dominated by zero entries reflecting the impossibility of most one period promotions like captain to colonel. Compensation (pay plus allowances) depends on grade level and is denoted by m_i, $i = 1, \ldots, k$. Furthermore, if an officer retires from grade level i, the fraction of military pay that is collected per period is given by r_i, the pension parameter, $0 \leq r_i \leq 1$. At each stage of the decision process (assume there are n + 1 stages remaining and that this is known for sure), an officer in state i may retire or remain in the Air Force.[2] If he stays, he receives compensation of amount m_i for this period and moves according to transition probability p_{ij} from state i to state j in the next period. Having arrived in state j we assume that he chooses the best decision and receives an optimal return of $V_n(j)$. The exact value of j is unknown, but the expected value of the optimal return at n is simply $\sum_{j=i}^{k} p_{ij} V_n(j)$. At n + 1, this

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[1]For ease of presentation we suppress t and enlarge the state space accordingly.

[2]In fact, this is not quite true in that retirement is not possible from all states. For now we ignore this complication.

return is discounted by β so that the total return from staying in and behaving optimally for the remaining n periods is:

$$m_i + \beta \sum_{j=i}^{k} p_{ij} V_n(j) \quad . \tag{1}$$

On the other hand, if the officer decides to retire from state i when n + 1 periods remain, he receives a retirement income of $r_i m_i$ each period and immediately begins searching in the civilian labor market. Search is assumed to proceed in optimal fashion with $S_{n+1}(i)$ denoting the optimal return from search when n + 1 periods remain and state i has been achieved in the Air Force.[1] In general, a different wage offer distribution, F_i, will be associated with each grade level i, the presumption being that there is a relationship between grade achieved and productivity in the civilian sector. For now we merely note that the expected discounted return from retiring now and searching optimally in the civilian sector is given by:

$$\sum_{j=1}^{n+1} \beta^{j-1} r_i m_i + S_{n+1}(i) \tag{2}$$

The optimal decision at n + 1 is obtained by choosing the maximum of (1) and (2). Thus, we have derived the following functional equation:

$$V_{n+1}(i) = \max \; [m_i + \beta \sum_{j=i}^{k} p_{ij} V_n(j); \sum_{j=0}^{n} \beta^j r_i m_i + S_{n+1}(i)] \tag{3}$$

where $V_{n+1}(i)$ is the expected discounted return when n + 1 periods remain and the decisionmaker (officer) is in state (grade level) i and follows an optimal retirement strategy.

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[1]For discussion of this finite horizon search model, see Lippman and McCall, pp. 166-171.

At first we thought that the optimal retirement policy would have a fairly simple structure. So far, this has not proved to be the case. For this reason, we decided to perform a numerical analysis of a modified version of (3). Search has been eliminated from this functional equation by replacing $S_{n+1}(i)$ with W (age, education), where W is the discounted expected civilian wages for a given age-education cohort. In addition to the elimination of search, note that Eq. (3) assumes that Air Force officers are risk neutral and have perfect information about promotion probabilities and civilian wages. Later we will assume that officers are risk averse and bring search back into the model.

4. NUMERICAL RESULTS

We conducted a detailed numerical analysis of this functional equation:

$$V_{n+1}(i) = \max \left[m_i + \beta \sum_{j=i}^{k} p_{ij}^{t} V_n(j);\ \sum_{j=0}^{n-1} \beta^j r_i m_i + W\ (\text{age, education})\right] \quad (4)$$

The analysis is unique in that it contains actual Air Force data for the promotion probabilities, p_{ij}; the military compensation, m_i; and the pension parameters, r_i. Data on civilian wages, W, were obtained from the Bureau of Census Current Population Survey. Unless stated otherwise, the discount rate, ρ, is set at .10.

At each stage of the process the officer evaluates (4) and either stays in the Air Force or leaves. The dynamic program calculates the return from each of these decisions. The computer program prints out the higher value of the return function, the optimal decision (stay or leave), and the difference between the return from the optimal and suboptimal decisions. The last calculation reveals the importance of making the correct decision. These calculations are made for captains, majors, lieutenant colonels, and colonels. Officers are also distinguished by source of commission (ROTC, academy, and OTS), by their flying status (rated and nonrated), and whether their commission is reserve or regular.

Our base case studies the optimal retirement behavior of ROTC, nonrated officers. The promotion probability matrix is based on data from fiscal year 1969. The military and civilian pay scales were also obtained for 1969. The retirement plan had the following features: if retirement occurs before twenty years of service, no retirement benefits are received; if retirement occurs at twenty years, the retiree receives 50 percent of base pay associated with the highest grade level achieved; for every year after twenty, the pension parameter is augmented by 2-1/2 percentage points up to a maximum of 75 percent at thirty years of service.

The Base Case

Using these data and this retirement plan, the optimal retirement policy for Air Force captains (regular and reserve) was to stay in the Air Force until promoted or forced out.[1] For a regular captain with six years of service, the discounted expected return from following an optimal policy was $143,000 and the difference between staying and leaving is $34,500. Making an incorrect decision at this point or being forced out is extremely costly.

The optimal retirement policy for majors[2] (reserve and regular) is to stay until they complete twenty years of service and then retire. For a regular major with nineteen years of service, the discounted expected return of following an optimal policy is $165,000 and the difference between staying and leaving is $53,000. After the 50 percent pension parameter has been attained (at twenty years of service) the difference between leaving (the optimal decision) and staying is relatively small, $850 after twenty years of service and $1,200 after twenty-one years of service. Consequently, while we should never observe a major quitting after nineteen years of service, we may very well see some staying in beyond twenty-two, the advantage to leaving being offset by factors not measured with our data.

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[1] Captains are mandatorily retired if they have not been promoted to major after seventeen years of service.

[2] Mandatory retirement occurs after twenty-two years of service for majors.

The optimal retirement policy for lieutenant colonels[1] is for regulars to stay until their twenty-fifth year of service and for reserves to stay until their twenty-second year of service. The difference between the optimal policies for regulars and reserves is that the former have a higher probability of being promoted to colonel. For a regular lieutenant colonel with twenty-four years of service, the discounted expected return of following an optimal policy is $186,000 and the difference between staying and leaving is $3,600. From twenty-two until twenty-seven years of service, the cost of making the wrong decision for regulars varies from $52 to $3,600. For most cases, the loss is less than $2,000. Other factors not measured by our data could cause lieutenant colonels in this age interval to make the "wrong" decision.

The optimal retirement policy for colonels[2] (regular and reserve) is to stay until they complete twenty-six years of service. For a colonel with twenty-five years of service, the discounted expected return from following an optimal policy is $201,000 and the difference between staying and leaving is $7,700. The cost of remaining in the Air Force from twenty-seven to twenty-nine years of service is relatively small, ranging between $200 and $1,100.

The actual retirement behavior of our base case is quite similar to that predicted by our model. Roughly speaking the optimal numerical results are at the median of observed retirement behavior. This gives us confidence in our model and also in its predictions about retirement behavior in response to changes in the retirement parameters.

Changes in Civilian Pay

We now study the effect on retirement of changes in civilian pay, all other parameters of the base case being held fixed.

When civilian pay is increased by $500 and $1,000 per year, captains continue to remain in the Air Force until they are promoted or discharged. The expected discounted return of remaining in the Air Force for sixteen years is $129,000 for the $500 increase and $132,500 for the

[1]Mandatory retirement occurs after twenty-eight years of service for lieutenant colonels.

[2]Mandatory retirement occurs after thirty years of service for colonels.

$1,000 increase. The cost of making the wrong decision and leaving at sixteen instead of staying is $12,300 for the $500 increase and $11,900 for the $1,000 increase.

With the same increases in civilian pay, majors leave after twenty years. The expected discounted return from taking such an action is $170,000 for the $500 increase and $173,000 for the $1,000 rise. The costs of making the wrong decision are $1,311 and $1,766, respectively.

Reserve lieutenant colonels leave after twenty-two years for both the $500 and $1,000 increases. The expected discounted return from this optimal behavior is $183,000 and $185,700, respectively. A wrong decision at this point costs $473 and $900, respectively. With a $500 ($1,000) increase regular lieutenant colonels leave after twenty-three (twenty-two) years of service. The expected discounted return from this retirement policy is $184,000 ($185,700), whereas the cost of an incorrect decision is $650 ($900).

For both pay increases colonels leave after twenty-six years. The expected discounted returns are $207,000 and $204,000, respectively. The costs of making the wrong decision are relatively small, $260 and $700, respectively. However, a wrong decision at twenty-five years of service imposes stiff penalties, $7,000 and $6,600, respectively.

Clearly, captains will continue to stay when annual civilian earnings are reduced by $500 and $1,000. The costs of making the wrong decision at sixteen years of service and leaving are substantial--$13,200 and $13,700, respectively.

With a $500 decrease in civilian earnings, majors continue to leave after twenty, but the cost of making an incorrect decision is slight, $402. Indeed, when civilian wages decrease by $1,000, it pays majors to stay in an extra year. However, the loss associated with leaving after twenty instead of staying is only $52.

Regular lieutenant colonels leave after twenty-five years when civilian earnings are lowered by $500 and $1,000. For the $1,000 decrease, the cost of making the wrong decision is only $210. The costs of making the wrong decision at twenty-five and twenty-seven years of service are also slight, $210 and $921, respectively. The optimal behavior for reserve lieutenant colonels is similar.

It pays colonels to stay until completing twenty-eight (twenty-nine) years of service when annual civilian earnings decline by \$500 (\$1,000). The cost of making the wrong decision is once again negligible, \$212 (\$189). Indeed, in both cases, the loss from an incorrect decision from twenty-six through twenty-nine years of service is small. However, the cost of an incorrect decision at twenty-five years is large, \$8,700 (\$10,200).

We next multiplied annual civilian pay by .7, .8, .9, 1.0, 1.1, 1.2, and 1.3 and observed changes in optimal retirement behavior. These optimal responses are summarized in Table 1, where we first report the optimal decision, then the expected discounted return associated with optimal behavior, and finally, the loss from making the wrong decision. Of course, multiplying by unity replicates the base case. As expected, departures increase as civilian earnings rise. Captains stay even when civilian earnings are multiplied by 1.3. The cost of making the wrong decision and leaving after eleven years is \$9,643 instead of \$15,899 when civilian earnings were multiplied by .7. Majors stay when earnings in the civilian sector are reduced by .7 and .8. The expected discounted return from this optimal strategy is \$137,366 and \$147,207, respectively. When civilian earnings increase to .9 of the base case, it pays majors to leave after twenty-one years. Notice that the cost of making a wrong decision here is negligible, \$88. When civilian earnings are multiplied by 1.3, the cost of not leaving after twenty years, the optimal decision, is no longer negligible, \$4,153. With one exception, the behavior of lieutenant colonels and colonels is as anticipated. The exception was the behavior of colonels when civilian earnings were multiplied by 1.3. The optimal behavior for this case was to leave after twenty-seven, leave after twenty-three, stay after twenty-four, stay after twenty-five, and leave after twenty-six. This illustrates a case in which a control limit rule of the forms, retire if $x \geq \xi$ and stay otherwise, is violated. Initially, we had conjectured that the optimal retirement policy would possess a control limit structure. This behavior provides a counterexample to this conjecture.

Table 1

OPTIMAL RESPONSE TO PROPORTIONAL CHANGES
IN ANNUAL CIVILIAN EARNINGS

Proportion of Base Case Civilian Earnings	Captain	Major	Lt. Colonel	Colonel
.7	Stay 95,153 15,899	Stay 137,360 2,117	Stay 154,996 1,488	Stay 173,863 2,206
.8	Stay 105,432 14,856	Stay 147,207 1,014	Stay 163,897 382	Stay 182,318 1,105
.9	Stay 115,711 13,813	lv. after 21 157,142 88	Stay 172,842 11	Stay 190,773 3
1.0	Stay 125,941 12,771	lv. after 20 166,757 857	lv. after 22 180,345 19	lv. after 27 200,923 1,098
1.1	Stay 136,270 11,728	lv. after 20 177,810 1,956	lv. after 22 191,177 1,124	lv. after 26 210,751 912
1.2	Stay 146,550 10,685	lv. after 20 188,864 3,054	lv. after 22 202,008 2,230	lv. after 26 220,956 2,020
1.3	Stay 156,829 9,643	lv. after 20 199,918 4,153	lv. after 20 208,043 2,857	[a]

[a]Nonmonotonic pattern.

Changes in Military Pay

Tables 2 and 3 present the optimal responses to changes in military pay. In Table 2, only the military pay is reduced to .7, .8, and .9 of its value in the base case. In Table 3, both military pay and military allowances are reduced by the same factors. The tables have the same format as Table 1 with the optimal decision presented first, then the expected discounted return from optimal behavior, and finally, the loss from taking the wrong action at the last decision point. In Table 2, colonels leave after twenty-six years even when their base pay is reduced by .7. However, the cost of making the wrong decision, staying an extra year, is increased from \$829 to \$2,876 as the factor of proportionality is decreased from .9 to .7.

In Table 3, lieutenant colonels leave after twenty-two years when pay and allowances are decreased by .9 and leave after twenty years when they are reduced by .7. For colonels the expected discounted return from behaving optimally (leaving after twenty-six years) diminishes from \$190,696 to \$170,998 as the factor of proportionality is reduced from .9 to .7.

Changes in the Discount Factor

Table 4 shows the changes in optimal retirement behavior as the discounted factor $\beta \equiv 1/1 + \rho$ changes. We investigated four different values .9524, .9302, .8889, and .8696 corresponding to discount rates, ρ, of .05, .075, .125, and .15, respectively. The format of the table is the same as its predecessors. In the base case the discount rate was equal to .10. As expected increases in the discount rate, ρ, causes Air Force officers to leave earlier, the value of the retirement plan diminishes. For example, when ρ = 5 percent (β = .9524), lieutenant colonels leave after twenty-six years. When ρ (β) increases (decreases) to 15 percent (.8696), lieutenant colonels leave after twenty-two years. Captains continue to stay for all values of β, but the expected discounted return decreases from \$179,805 to \$94,206 as β decreases from .9524 to .8696.

Table 2

OPTIMAL RESPONSE TO PROPORTIONAL CHANGES
IN ANNUAL MILITARY PAY

Proportion of Base Case Military Earnings	Captain	Major	Lt. Colonel	Colonel
	Stay	lv. after 20	lv. after 20	lv. after 26
.7	122,620	146,659	152,137	166,523
	9,399	3,727	2,808	2,876
.8	Stay	lv. after 20	lv. after 22	lv. after 26
	123,743	153,358	163,455	177,864
	10,523	2,770	2,082	1,853
	Stay	lv. after 20	lv. after 22	lv. after 26
.9	124,867	160,058	171,900	189,205
	11,647	1,813	1,050	829

Table 3

OPTIMAL RESPONSE TO PROPORTIONAL CHANGES
IN ANNUAL MILITARY PAY AND ALLOWANCES

Proportion of Base Case Military Earnings	Captain	Major	Lt. Colonel	Colonel
	Stay	lv. after 20	lv. after 20	lv. after 26
.7	122,620	149,891	155,579	170,998
	9,399	3,896	2,989	3,186
	Stay	lv. after 20	lv. after 20	lv. after 26
.8	123,743	155,513	162,013	180,847
	10,523	2,883	1,846	2,059
	Stay	lv. after 20	lv. after 22	lv. after 26
.9	124,867	161,135	173,142	190,696
	11,647	1,870	1,123	932

Table 4

OPTIMAL RESPONSE TO CHANGES IN
THE DISCOUNT FACTOR

β	Captain	Major	Lt. Colonel	Colonel
	Stay	Stay	lv. after 26	Stay
.9524	179,805	239,572	241,941	251,771
	13,379	253	573	285
	Stay	lv. after 20	lv. after 24	lv. after 28
.9302	149,146	198,400	211,524	225,482
	13,067	259	137	43
	Stay	lv. after 20	lv. after 23	lv. after 26
.8889	108,133	142,466	157,764	177,698
	12,487	1,259	845	317
	Stay	lv. after 20	lv. after 22	lv. after 26
.8696	94,206	123,467	136,077	159,302
	12,216	1,535	830	690

The Retirement Modernization Act

We now study the effects of implementing the Retirement Modernization Act. The pension parameter, r, is now calculated according to:

$$r = .025 \min (YOS, 24) + .03 \max (0, YOS - 24)$$

where YOS denotes years of service. If YOS at retirement is less than thirty, .15 is subtracted from r until YOS plus time since retirement equals thirty at which point the .15 is restored. As before $r = 0$, if $YOS < 20$.

Changes in Civilian Pay

Table 5 is identical to Table 1 except that now the effects of proportional variations in civilian pay are measured after implementing the Retirement Modernization Act. It is informative to compare the base case with the Retirement Modernization Act, the former corresponding to a unitary factor of proportionality in Table 1 while the latter corresponds to a unitary factor of proportionality in Table 5. For the base case, majors leave after twenty; lieutenant colonels after twenty-two; and colonels after twenty-seven. For the Retirement Modernization Act everyone stays.

Changes in Military Pay

Tables 6 and 7 display the optimal retirement response to changes in military pay after implementation of the Retirement Modernization Act. They correspond to Tables 2 and 3 for the base case.

Changes in the Discount Factor

Table 8 presents changes in optimal retirement behavior as a function of the discount factor. It is similar to Table 4, the only difference being that we are now evaluating the Retirement Modernization Act instead of the base case.

Table 5

OPTIMAL RESPONSE TO PROPORTIONAL CHANGES
IN ANNUAL CIVILIAN EARNINGS
(RETIREMENT MODERNIZATION ACT)

Proportion of Base Case Civilian Earnings	Captain	Major	Lt. Colonel	Colonel
	Stay	Stay	Stay	Stay
.7	95,153	128,570	153,825	177,727
	15,899	3,765	3,796	4,813
	Stay	Stay	Stay	Stay
.8	105,432	138,417	162,727	186,182
	14,856	2,662	2,640	3,712
	Stay	Stay	Stay	Stay
.9	115,711	148,265	171,629	194,637
	13,813	1,559	1,584	2,610
	Stay	Stay	Stay	Stay
1.0	125,991	158,112	180,531	203,092
	12,771	456	478	1,509
	Stay	lv. after 20	lv. after 26	Stay
1.1	136,270	166,675	188,222	211,547
	11,728	308	202	407
	Stay	lv. after 20	lv. after 22	lv. after 28
1.2	146,550	197,729	190,569	270,696
	10,686	1,407	281	694
	Stay	lv. after 20	lv. after 22	lv. after 26
1.3	156,829	188,782	201,401	22,464
	9,643	2,506	1,387	250

Table 6

OPTIMAL RESPONSE TO PROPORTIONAL CHANGES
IN ANNUAL MILITARY PAY (RETIREMENT MODERNIZATION ACT)

Proportion of Base Case Military Earnings	Captain	Major	Lt. Colonel	Colonel
	Stay[a]	lv. after 20	lv. after 20	lv. after 26
.7	122,620	139,504	143,897	162,139
	9,399	2,668	1,589	993
	Stay	lv. after 20	lv. after 22	lv. after 27
.8	123,743	144,877	154,199	174,465
	10,523	1,515	589	35
	Stay	lv. after 20	lv. after 26	lv. after 29
.9	124,867	150,249	169,110	189,716
	11,647	362	236	340

[a]Optimal decision is to leave after four, five, six, and seven years. However, if you are in at eight years then optimal to stay until promotion or discharge.

Table 7

OPTIMAL RESPONSE TO PROPORTIONAL CHANGES IN ANNUAL MILITARY PAY AND ALLOWANCES (RETIREMENT MODERNIZATION ACT)

Proportion of Base Case Military Earnings	Captain	Major	Lt. Colonel	Colonel
	Stay[a]	lv. after 20	lv. after 20	lv. after 26
.7	122,620	142,096	146,657	166,310
	9,399	2,743	1,669	1,172
	Stay	lv. after 20	lv. after 22	lv. after 27
.8	123,743	146,605	156,788	177,392
	10,523	1,565	667	170
	Stay	lv. after 20	lv. after 26	Stay
.9	124,867	151,113	170,420	191,238
	11,647	387	292	256

[a]Optimal decision is to leave after four, five, six, and seven years. However, if you are in at eight years, then the optimal policy is to stay until promoted or discharged.

Table 8

OPTIMAL RESPONSE TO CHANGES IN
THE DISCOUNT FACTOR, β (RETIREMENT MODERNIZATION ACT)

β	Captain	Major	Lt. Colonel	Colonel
	Stay	Stay	Stay	Stay
.9524	179,805	228,416	236,166	257,541
	13,379	1,978	2,114	3,303
	Stay	Stay	Stay	Stay
.9302	149,146	188,122	204,693	226,622
	13,067	1,066	1,158	2,267
	Stay	Stay	Stay	Stay
.8889	108,133	135,019	161,395	184,668
	12,487	34	18	940
	Stay	lv. after 21	lv. after 26	Stay
.8696	94,206	117,172	141,774	169,986
	12,216	265	5	509

5. FURTHER RESEARCH

There are several directions in which we plan to expand the research reported here. Some of this will be included in revised versions of this paper, the remainder being postponed until later papers.

We will test the implications of the dynamic retirement model using standard statistical techniques. The model will be modified to include leisure in the post-Air Force employment decision.

Air Force officers have been assumed to have linear utility functions. We will drop this and allow them to have concave utility functions. The behavioral implications of risk aversion will then be studied. Having introduced risk aversion, we will also include search explicitly in the retirement model, i.e., we will return to the initial formulation of the retirement model. We will also distinguish between on-the-job search and post-retirement search.

The effect of the business cycle on retirement decision will be investigated more thoroughly than was done here. The retirement decision will be embedded in a steady state model which will allow us to say something about improvements as a consequence of retirement policy changes.

III. WELFARE PARTICIPATION MODELS

1. INTRODUCTION

This section presents several simple models of welfare participation. The first highlights the stochastic nature of welfare dependency. It is an elementary model of movements on and off welfare. Both the incidence and duration of welfare are exogenous to the individual household. Furthermore, there is no choice among alternative income transfer programs--welfare is the only income transfer program and participation begins as soon as a family becomes eligible. The second model introduces choice into the naive stochastic model. The population is composed of male-headed households who may choose among several actions when they become eligible for income transfers. The actions are: to participate in the Unemployment Insurance (UI) program; to participate in the AFDC-Unemployed Fathers (UF) program; and to participate in neither.

The choice between participating in either the AFDC-UF welfare program or the UI unemployment compensation system is accomplished by calculating the net benefit of each. If welfare provides greater (less) net benefits, then UF (UI) is the preferred transfer program. This simple observation is complicated by two factors.

First, the net benefits that we will calculate here assume that the decisionmaker behaves optimally with respect to the chosen transfer program. The most important decision to be made after choosing the preferred transfer program is determining the level of job search. We assume that this decision is also made in optimal fashion.

The second complication is that the net benefits from an optimal program choice depend on the reason for unemployment. Here we distinguish four different kinds of unemployment. The first is a layoff with a zero probability of recall. The second is a layoff with a nonzero probability of recall and with more than one recall offer. The third is a layoff with a nonzero probability of recall where there is only one recall offer. All three of these cases are analyzed in an infinite horizon setting with discounting. The fourth model takes

cognizance of the finite time span *and* the fact that unemployment insurance is available only for a fixed number of periods. All of these models are limited in that: (a) households are assumed to be risk indifferent; (b) households are assumed to know their wage offer distributions as well as the rules and regulations governing the income transfer programs; and (c) households know the costs of search and the cost of participating in each transfer program. Probably the major limitation is the complexity of both the unemployment compensation system and the welfare program. The eligibility requirements for each program are quite different and are themselves highly variable both in statement and enforcement over time and across states. A clear understanding of these institutional arrangements is a necessary prelude to any hypothesis testing and questionnaire design.

2. A STOCHASTIC MODEL OF WELFARE PARTICIPATION

This section is a reminder that welfare participation is affected by probabilistic considerations. To emphasize this and to motivate the subsequent sections *only* stochastic properties are examined. Each household is engaged in a continuous time random walk (a birth and death process) over which it has no control. Households who are not on welfare achieve eligibility according to a Poisson process with parameter λ. In this simple birth and death process (infinite server queue) there is no delay between achieving eligibility and acceptance into the welfare program. The time on welfare is assumed to be an exponential random variable with parameter μ. It follows that the transition law governing this process can be described as follows: if there are k individuals on welfare at time t (k is the state variable), the probability of $k + 1$ at $t + \Delta t$ is given by $\lambda\Delta t$ plus terms whose sum is negligible as Δt goes to zero; the probability of $k - 1$ at $t + \Delta t$ is $\mu\Delta t$ plus terms whose sum is negligible; the probability of k at $t + \Delta t$ is $1 - \lambda\Delta t - \mu\Delta t$ plus negligible terms. This transition law gives rise to a set of differential equations that completely describe the dynamics of this birth and death process.[1] In particular,

[1]For a nice discussion of birth and death processes, see Karlin.

the following key transition probability can be calculated from these differential equations. Let n be the number of households on welfare at time t = 0. The probability of having m households on welfare at some t > 0 converges to

$$P_{nm}(t) = \frac{(\lambda/\mu)^m e^{-\lambda/\mu}}{m!}$$

as t goes to infinity.

A model like this might not be a bad description of welfare dynamics for a homogeneous group (same λ) with exponential time on welfare.[1] However, it completely ignores the economic behavior of households. This is partially remedied in the subsequent sector by allowing eligible households to choose among alternative income transfer programs. It is only a palliative in that the incentive effects of the welfare program on the number who become eligible is ignored.

3. PARTICIPATION MODELS

The choice between UI and UF is analyzed in this section. We first develop optimal policies for UI participation as a function of the reason for unemployment. The return associated with each of these policies is compared with the return from entering welfare. The models of welfare participation also assume optimal search behavior on the part of the unemployed father. They also explicitly recognize the fixed cost of going on welfare and the additional costs of job search while on welfare. That is, we assume that the costs of welfare qualification are much higher than those of unemployment insurance *and* that other things equal employers prefer to hire nonwelfare recipients. It is these entry and exit costs that perhaps are the most important manifestation of the welfare stigma.

Let us first consider the choice confronting an individual who has just been fired with zero probability of recall to his former job. For simplicity, we assume that he is eligible for both UI and UF.

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[1] See Boskin and Nold, Rydell, et al., and Saks for alternative views on the distribution of time on welfare.

Each period of unemployment he receives unemployment compensation of amount u if he chooses UI and welfare payments of amount w if he chooses UF. In most states, u is independent of family size and is not reduced as the earnings of other family members increase. On the other hand, w is reduced as the earnings of other family members increase and is positively related to family size. The welfare payment also includes in-kind benefits like Medicaid, food stamps, and housing allowances. The simple model of choice presented here ignores these problems. Their incorporation which will be attempted later should not alter the basic structure of the optimal policy. Finally, for analytical simplicity the decision models are posed in an infinite horizon setting with risk neutrality and perfect information about program features and wage distributions.

Let $V_I(x)$ be the maximum discounted expected benefits from the UI program when a job offer x has just been received. It follows that V_I satisfies the following recursive relation:

$$\begin{aligned} V_I(x) &= \max\,[x,\ (u - c_I) + \beta \textstyle\int_0^\infty V(y)dF(y)] \\ &= \max\,[x,\ \xi_I]\ , \end{aligned} \tag{1}$$

where c_I is the per-period cost of search under UI, β is the discount factor, and F is the cumulative distribution function of wage offers for this individual. It is well-known that the optimal policy is characterized by the single number ξ_I such that

if $x \geq \xi_I$ accept employment

if $x < \xi_I$ continue search .

The optimal return from this policy is ξ_I. This return is an increasing function of u. Hence, an immediate consequence of this analysis is that the greater the unemployment compensation, the longer the period of unemployment.

Having calculated ξ_I the decisionmaker will compare it with the optimal return from welfare participation. Let $V_w(x)$ be the maximum discounted expected benefits from the UF program when a job offer x has just been received. Arguing as before V_w satisfies:

$$V_w(x) = \max\,[x,\ (w - c_w) + \beta \int_0^\infty V(y)dG(y)]$$
$$= \max\,(x,\ \xi_w) \quad . \tag{2}$$

The optimal policy has the same structure as before with ξ_w, the optimal return, replacing ξ_I. By the previous argument we expect c_w, the cost of search from welfare, to exceed c_I. Furthermore, the wage offer distribution, G, may be stochastically smaller than F, i.e., $F(t) \leq G(t)$, for all t. Both of these will tend to lower ξ_w. As before, the higher w, the larger ξ_w.

The decisionmaker will choose UI if

$$\frac{\xi_I}{1 - \beta} > \frac{\xi_w}{1 - \beta} - K_w \tag{3}$$

where K_w is the fixed cost of going on welfare, and $\xi/1 - \beta$ is the present value of getting ξ dollars in all future periods. Of course, in this infinite horizon model it may be optimal to refrain from search and remain on welfare or unemployment insurance indefinitely. This will be true if

$$\max\,[\frac{w}{1 - \beta}, \frac{u}{1 - \beta}] > \max\,[\frac{\xi_I}{1 - \beta}, \frac{\xi_w}{1 - \beta} - K_w] \quad .$$

There has been considerable discussion of the cumulative distribution function, F, for the random variable, τ, time on welfare. Some have argued that is is exponential with the closing rate,

$$c(t) = \frac{f(t)}{1 - F(t)} \quad ,$$

a constant, while others maintain that the closing rate decreases with time. Clearly, the time on welfare is determined by the cost of search, the size of the welfare payment, the job offer distribution, and tax rates. All of these factors will be summarized by the reservation wage, ξ_w, when it is calculated properly. The time till an offer exceeds ξ_w is a geometric random variable with a constant closing rate. Of course, the job offer distribution is fluctuating with the business cycle; welfare tax rates are changing as are other aspects of welfare policy. Variations like these will cause ξ_w to change over time. Thus, while a static model predicts a constant closure rate, it is not clear on theoretical grounds whether the actual closing rate is increasing or decreasing. However, we can sign the changes in the closing rate as a function of the cycle, welfare rules, etc. All of the preceding discussion assumes that welfare recipients have been partitioned into homogeneous classes. If this has not been accomplished, the resulting heterogeneity will also cause the closure rate to vary.

The choice confronting an individual who has been laid off, but has a nonzero probability of recall, can be analyzed in a similar fashion. Here, however, there are two cases to be distinguished. In the first, the firm instituting the layoff will submit more than one offer to its former employee, i.e., the former employee may reject an offer from its former firm and remain eligible for further offers. In the second, the former firm presents an offer which if rejected disqualifies the laid off employee from further consideration.

In the first case, let $V_I(x)$ be the maximum discounted expected benefits from UI when a job offer x has just been received. V_I satisfies the following functional equation:

$$\begin{aligned} V_I(x) &= \max\,[x,\ -c_I + u + p\beta \textstyle\int_0^\infty \int_0^\infty \max\,(V_I(y),\ V_I(z)) \\ &\qquad dF(y)dG(z) + (1-p)\,\beta \textstyle\int_0^\infty V_I(y)dF(y)] \qquad (4) \\ &= \max\,[x,\ \xi_i^r] \end{aligned}$$

where p is the probability of recall by his former employee, F is the offer distribution from other firms, G is the offer distribution from the former firm. (Usually G will be stochastically larger than F because of specific human capital considerations. However, it is certainly possible that adverse information possessed by the former employee could cause F to be stochastically larger than G.) Thus, if a recall occurs, the individual must compare the return from accepting that offer with the return from accepting an offer from another firm. The preferable alternative is then compared with the option to continue search. If no recall occurs, the return from continuing search is compared with the return from accepting employment. The optimal return from following this policy is ξ_I^r. A similar argument leads to ξ_w^r, the optimal return from participating in the UF program when the probability of recall is nonzero.

The individual also may choose not to search at all and simply wait for a recall.[1] Assuming that the recall wage r exceeds u(w), it is easy to show that the expected discounted return from this option exceeds $u/1 - \beta(w/1 - \beta)$.[2] Thus, there are several options: UI (UF) with search, UI (UF) without search. UI with search is preferable to UF if:

$$\frac{\xi^r}{1 - \beta} > \frac{\xi_w^r}{1 - \beta} - K_w \quad . \tag{5}$$

UI (UF) without search is preferable to UI (UF) with search if $u(w) > \xi_I^r(\xi_w^r)$. Since $u(w)/1 - \beta$ is a lower bound on the return from UI (UF) without search, this alternative may be preferable even if $u(w) < \xi_I^r(\xi_w^r)$.

A similar type of analysis can be performed when the layoff firm submits only one offer--the take it or leave it case. The functional equation for the UI participant is given by:

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[1] See Feldstein.

[2] We continue to assume that u(w) is paid during every period of unemployment.

$$V_I(x) = \max [x, -c_I + u + p\beta \int_0^\infty U_I(z)\, dH(z) + (1 - p)\, \beta \int_0^\infty V_I(y)\, dF(y)] = \max (x, \xi_V) \tag{6}$$

where $z = \max (x, y)$ with x the value of the offer from F and y the value of the offer from G. The random variable Z has distribution H. Finally, $U_I(x)$ satisfies:

$$U_I(x) = \max [x, -c_I + u + \int_0^\infty U_I(y)\, dF(y)] = \max (x, \xi_u) \quad . \tag{7}$$

If the offer from the former firm is rejected, the situation is exactly that of the unemployed worker with zero probability of recall. Clearly, $V \geq U$ which implies that $\xi_V \geq \xi_u$. Comparisons with welfare and waiting till the offer from the former firm are analogous to the case of multiple offers.

We now present a finite horizon model in which unemployment insurance is available only for a fixed number of periods. Let N be the number of periods remaining till retirement and n the number of periods of unemployment compensation. The choice between UI and UF can be made by analyzing the following functional equation:

$$V_{N,n}(x) = \max [x; - c_I + u + \beta \int_0^\infty V_{N-1,n-1}(y) dF(y); -c_w + w + \beta \int_0^\infty V_{N-1,n}(y) dG(y)] \quad . \tag{8}$$

It seems clear that if UI is preferred to UF for any n then it is preferred for all n, i.e., a switch to UF will never occur until UI benefits are exhausted. However, if F deteriorates as time goes by, the switch to UF could occur before exhaustion. This merits further study. In particular, it would be very instructive to conduct a numerical analysis of this functional equation as was done for the retirement model.

4. CONCLUDING COMMENTS

Individuals who quit into unemployment face roughly the same analytical problem as those who were permanently discharged. The two groups are, however, likely to be quite different. The former may be composed of younger individuals whose job expectations were not met, but yet were not able to make another job attachment before quitting. It should be more difficult for those who were permanently discharged to obtain employment. Thus, the offer distributions facing these two groups should differ. Finally, the quitters may not be immediately eligible for UI. Having said all of this, quitters will choose between UI and UF by checking inequality (3).

Those who have exhausted their UI benefits will choose between continued search on their own (Eq. (1) with u = 0 and c_I replaced by c_E, the cost per-period of exhaustee search) and UF. Letting ξ_E denote the optimal return from searching on their own, UF is preferable if

$$\frac{\xi_w}{1-\beta} > \frac{\xi_E}{1-\beta} + K_w \quad .$$

In calculating the returns from welfare participation, we assumed that the unemployed fathers would engage in full time search. In practice, they may have the choice between full time search, part-time search, and no search at all. Our analysis can easily incorporate these alternatives.

When families choose to go on welfare, they may also consider the alternative of family dissolution with the mother and children participating in a regular AFDC program and the father living on his own. The mother will decide whether full time search, part-time search, or no search is best. The return from this decision will be compared with the return they were getting under the UF program. Family dissolution will occur when the comparison becomes sufficiently unfavorable for UF. In our analysis here, we have ignored the effects

of imperfect information on the returns to UF participation. Thus, family dissolution would occur before UF was begun. In practice, of course, the unemployed father will be revising his estimate of the return from the UF program after he enters UF. If the demand for his labor is less than he expected, the return from UF will be revised *downward*. Also, because of the finite time horizon, the longer he searches the lower the returns from search. These effects, when coupled with the likelihood that welfare costs increase for the father the longer he is on UF, could account for the large incidence of family dissolution among the UF cases.

REFERENCES

Boskin, M. J. and F. C. Nold, "A Markov Model of Turnover in Aid to Families with Dependent Children," *Journal of Human Resources*, Fall 1975.

Cotterman, R., *A Dynamic Theory of Retirement*, unpublished, University of Chicago, 1975.

Feldstein, M., "Temporary Layoffs in the Theory of Unemployment," *Journal of Political Economy*, October 1976.

Karlin, S., *A First Course in Stochastic Processes*, Academic Press, 1966.

Lippman, S and J. McCall, "The Economics of Job Search: A Survey," *Economic Inquiry*, September 1976.

Rydell, C. P., T. Palmerio, G. Blais, and D. Brown, *Welfare Case Dynamics in New York City*, Santa Monica, Calif., The Rand Corporation, R-1441-NYC, October 1974.

Saks, D. H., *Public Assistance for Mothers in an Urban Labor Market*, Industrial Relations Section, Princeton University, 1975.